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Simple Radiative Neutrino Mass Matrix for Solar and Atmospheric Oscillations

Ernest Ma

Department of Physics
University of California
Riverside, California 92521

Abstract

A simple 3×3 neutrino Majorana mass matrix is proposed to accommodate both the solar and atmospheric neutrino deficits. This scenario can be realized naturally by a radiative mechanism for the generation of neutrino masses. It also goes together naturally with electroweak baryogenesis and cold dark matter in a specific model.

There is now a good deal of evidence from different experiments that there exists a solar neutrino deficit[1, 2, 3, 4] as well as mounting evidence for an atmospheric neutrino deficit.[5, 6] In terms of neutrino oscillations, the former (latter) is an indication that ν_e (ν_μ) is not a mass eigenstate.[7, 8] A popular approach to the neutrino-mass problem is the seesaw mechanism,[9] in which case m_{ν_l} is naively expected to be proportional to m_l^2, where $l = e, \mu, \tau$, and the mixing angles are assumed to be small, in analogy with what is observed in the quark sector. However, that is not the only, nor necessarily the most natural, possibility. In this paper, a very different form of the neutrino mass matrix will be proposed. It is simple and can be realized naturally by a radiative mechanism for the generation of Majorana neutrino masses. It also fits very well into the framework of a recently proposed doublet Majoron model[10, 11] which allows for the generation of baryon number during the electroweak phase transition as well as having ν_τ as the late decaying particle for a consistent interpretation that the missing mass of the Universe is all cold dark matter.

Consider the following 3×3 Majorana mass matrix for the states ν_e, ν_μ, and ν_τ (or ν_S, a hypothetical singlet neutrino):

$$\mathcal{M}_\nu = \left[\begin{array}{ccc} \epsilon_1 & \epsilon_4 & m\cos\theta \\ \epsilon_4 & \epsilon_2 & m\sin\theta \\ m\cos\theta & m\sin\theta & \epsilon_3 \end{array} \right], \tag{1}$$

where

$$\epsilon_{1,2,3,4} << m. \tag{2}$$

Let the mass eigenstates be denoted by $n_{1,2,3}$, then the corresponding mass eigenvalues are

$$m_1 \simeq m + \frac{1}{2}(\epsilon_1 \cos^2\theta + \epsilon_2 \sin^2\theta + \epsilon_3 + \epsilon_4 \sin 2\theta), \tag{3}$$

$$m_2 \simeq -m + \frac{1}{2}(\epsilon_1 \cos^2\theta + \epsilon_2 \sin^2\theta + \epsilon_3 + \epsilon_4 \sin 2\theta), \tag{4}$$

$$m_3 \simeq \epsilon_1 \sin^2\theta + \epsilon_2 \cos^2\theta - \epsilon_4 \sin 2\theta, \tag{5}$$

and

$$\nu_e \simeq \frac{1}{\sqrt{2}}(n_1 - n_2)\cos\theta - n_3 \sin\theta, \tag{6}$$

$$\nu_\mu \simeq n_3 \cos\theta + \frac{1}{\sqrt{2}}(n_1 - n_2)\sin\theta, \tag{7}$$

$$\nu_\tau(\nu_S) \simeq \frac{1}{\sqrt{2}}(n_1 + n_2). \tag{8}$$

From Eqs. (3) - (5), we see that

$$\begin{aligned} \Delta m_{12}^2 &\simeq 2m(\epsilon_1 \cos^2\theta + \epsilon_2 \sin^2\theta + \epsilon_3 + \epsilon_4 \sin 2\theta) \\ &<< m^2 \simeq \Delta m_{13}^2 \simeq \Delta m_{23}^2. \end{aligned} \tag{9}$$

This means that $\nu_\mu - \nu_e$ oscillations are governed by m^2 and $\sin^2 2\theta$, which can be chosen to be about $10^{-2}\mathrm{eV}^2$ and 0.5 respectively[12] to account for the atmospheric neutrino data.[5, 6] As for the solar neutrino deficit, the ν_e flux is first diminished by its rapid oscillation into ν_μ to $(1 - \frac{1}{2}\sin^2 2\theta)$ of its initial value, then the oscillation into ν_τ (or ν_S) with $\sin^2 2\theta_{12} = 1$ and Δm_{12}^2 of about $10^{-10}\mathrm{eV}^2$ for the vacuum oscillation solution[13] reduces it further[14] to what is observed.[1, 2, 3, 4] Matter-enhanced oscillations[15] are not possible here because the mixing is maximum, *i.e.* $\theta_{12} = \pi/4$.

The above discussion shows that as long as the $\nu_e - \nu_\tau$ (or $\nu_e - \nu_S$) and $\nu_\mu - \nu_\tau$ (or $\nu_\mu - \nu_S$) entries of the 3×3 Majorana neutrino mass matrix are much greater than all other entries, the resulting mass eigenstates will be such that a linear combination of ν_e and ν_μ pairs up with ν_τ (or ν_S) to form a pseudo-Dirac neutrino, *i.e.* an equal (or almost equal) admixture of two nearly degenerate Majorana neutrinos. With suitable values for the two large entries and a general magnitude for the small ones, both the solar and atmospheric neutrino deficits are explained. The question now is whether such a simple ansatz has a natural realization. It may be of interest to note that in the discredited case of the 17-keV neutrino, the most probable theoretical explanation was that a linear combination of ν_e and ν_τ pairs up with ν_μ to form a pseudo-Dirac neutrino.[16]

Since m and Δm^2_{12} should be of order 0.1 eV and $10^{-10}\mathrm{eV}^2$ respectively, the ratios $\epsilon_{1,2,3,4}/m$ are of order 10^{-8}. Hence it is natural to assume as a first approximation that $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0$. This can be achieved by the imposition of a discrete symmetry which is then softly broken so that $\epsilon_{1,2,3,4}$ may acquire small nonzero values. Since m itself is already rather small, a natural explanation is that of radiative generation.[17] In the following it will be shown how everything can be done in the context of the recently proposed doublet Majoron model.[10, 11]

If there is no ν_S and $\mathcal{M}_\nu$ refers to the known three light neutrinos, then they have no impact on the question of dark matter in the Universe because the sum of their masses would be much less than 1 eV. After the results of the Cosmic Background Explorer (COBE),[18] it is popularly assumed that the Universe contains 70% cold dark matter and 30% hot dark matter.[19] The latter could be neutrinos, but the sum of their masses has to be about 7 eV. Implications of this assumption on the neutrino mass matrix have been explored.[20] On the other hand, it is also possible that the Universe contains 100% cold dark matter and the COBE results are explained by a late decaying particle,[11, 21, 22, 23, 24] the prime candidate being ν_τ, but its mass should be a few MeV. There is actually another good reason for a ν_τ of this mass. Its Yukawa coupling would then be large enough to allow for the possible generation of the observed baryon-number asymmetry of the Universe during the electroweak phase transition from the spontaneous breaking of lepton-number conservation.[25] This mechanism requires a detailed understanding of transmission through and reflection off bubble walls, and is under active investigation.[26]

The recently proposed doublet Majoron model[10, 11] provides a natural framework for both electroweak baryogenesis and cold dark matter. Since m_{ν_τ} is a few MeV in this case, the mass matrix $\mathcal{M}_\nu$ of Eq. (1) should now be interpreted as representing ν_e, ν_μ, and ν_S, the last being a singlet neutrino, each having lepton number $L = 1$. Note that in this model,[10, 11]

lepton number corresponds to a conserved global U(1) symmetry above the energy scale of electroweak symmetry breaking. It is broken spontaneously together with the SU(2) $\times$ U(1) gauge symmetry necessarily and a lepton asymmetry of the Universe is created which gets converted into a baryon asymmetry through sphalerons.[25] The massless Goldstone boson associated with the spontaneous breaking of L is called the Majoron. The massive ν_τ's annihilate into Majorons very quickly in this model so that the ν_τ contribution to the energy density of the Universe at the time of nucleosynthesis is negligible. On the other hand, ν_τ decays rather slowly and as the Universe expands, it eventually becomes dominant, but only until it finally decays away into Majorons and other light neutrinos. This scenario is thus very much suited for the radiative generation of Majorana neutrino masses[17] because lepton number is already assumed to be spontaneously broken.

In addition to all the particles of the standard model, let there be one light singlet neutrino ν_{SL} with $L = 1$, one heavy neutral singlet fermion N_R with $L = 0$, and two scalar doublets $\Phi_{1,2} = (\phi^+_{1,2}, \phi^0_{1,2})$ with $L = \mp 1$. To obtain $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0$ in $\mathcal{M}_\nu$, assume a discrete Z_3 symmetry such that $(\nu_e, e)_L, (\nu_\mu, \mu)_L, e_R, \mu_R$ transform as ω, whereas $(\nu_\tau, \tau)_L, \tau_R, \nu_{SL}, N_R$ transform as ω^2, with $\omega^3 = 1$. To obtain radiative neutrino masses, assume the existence of three charged scalar singlets $\eta^-_{0,1,2}$ with $L = 0, 1, 2$ respectively. All scalar particles are assumed to be trivial under Z_3. As a result, the $\nu_e - \nu_S$ and $\nu_\mu - \nu_S$ mass terms are generated in one loop as shown in Fig. 1, but all other entries of $\mathcal{M}_\nu$ remain zero. Specifically,

$$m \cos\theta = \frac{f_{e\tau} m_\tau f_{\tau S}}{16\pi^2} \frac{v_1^2 v_0^2}{M^4}, \tag{10}$$

$$m \sin\theta = \frac{f_{\mu\tau} m_\tau f_{\tau S}}{16\pi^2} \frac{v_1^2 v_0^2}{M^4}, \tag{11}$$

where $v_{0,1}$ are the vacuum expectation values of $\phi^0_{0,1}$, and M is an effective mass of the η's in the loop. However, $\mathcal{M}_\nu$ is only a submatrix of a larger 5×5 matrix containing also ν_τ and N. Assuming a heavy Majorana mass for N (which breaks Z_3 softly), ν_τ gets a seesaw mass due to its coupling to N via ϕ^0_1. The effective 4×4 mass matrix spanning ν_e, ν_μ, ν_S,

and ν_τ is then given by

$$\mathcal{M}'_\nu = \begin{bmatrix} 0 & 0 & m\cos\theta & m'\cos\theta' \\ 0 & 0 & m\sin\theta & m'\sin\theta' \\ m\cos\theta & m\sin\theta & 0 & 0 \\ m'\cos\theta' & m'\sin\theta' & 0 & m_{\nu_\tau} \end{bmatrix}. \tag{12}$$

The $\nu_e - \nu_\tau$ and $\nu_\mu - \nu_\tau$ mass terms are also radiatively induced in one loop as in Fig. 1, but with ν_S replaced by ν_τ and η_0^- by ϕ_0^-. As a result,

$$m'\cos\theta' = \frac{f_{e\tau}(m_\tau^2 - m_e^2)}{16\pi^2}\frac{\Lambda v_1^2}{M^4}, \tag{13}$$

$$m'\sin\theta' = \frac{f_{\mu\tau}(m_\tau^2 - m_\mu^2)}{16\pi^2}\frac{\Lambda v_1^2}{M^4}, \tag{14}$$

where Λ is the cubic $\phi_0^+\phi_1^0\eta_1^-$ coupling. Comparing Eqs. (13) and (14) to Eqs. (10) and (11), it is clear that $\theta \simeq \theta'$, and $\sin^2 2\theta = 0.5$ is obtained if $f_{\mu\tau}/f_{e\tau} = 0.4$. Using $M = 1$ TeV, $v_0 = 245$ GeV, and $v_1 = 22$ GeV, a value of 0.1 eV for m is also obtained if $\sqrt{f_{e\tau}^2 + f_{\mu\tau}^2} = 0.01$ and $f_{\tau S} = 0.03$. Because of mixing with ν_τ, the effective $\mathcal{M}_\nu$ of Eq. (1) now has

$$\epsilon_1 \simeq m'^2\cos^2\theta/m_{\nu_\tau}, \qquad \epsilon_2 \simeq m'^2\sin^2\theta/m_{\nu_\tau}, \tag{15}$$

$$\epsilon_3 = 0, \qquad \epsilon_4 \simeq m'^2\sin\theta\cos\theta/m_{\nu_\tau}. \tag{16}$$

Therefore,

$$\Delta m_{12}^2 \simeq 2mm'^2/m_{\nu_\tau}. \tag{17}$$

Using $\Lambda = 400$ GeV, a value of about 0.04 eV for m' is obtained. Hence $\Delta m_{12}^2 \simeq 10^{-10}\mathrm{eV}^2$ if $m_{\nu_\tau} \simeq 3$ MeV. These numbers clearly demonstrate that a natural radiative realization of $\mathcal{M}_\nu$ is possible for a successful explanation of the solar and atmospheric neutrino deficits. It should be mentioned that $\mathcal{M}'_\nu$ of Eq. (12) has also been obtained with a Dirac seesaw mechanism in a recently proposed singlet-triplet Majoron model.[24]

Consider now the decay of ν_τ in the present model. It proceeds via the mixing of ν_e and ν_μ with ν_τ in $\mathcal{M}'_\nu$ which is m'/m_{ν_τ}. The rate is given by[11]

$$\Gamma = \frac{m'^2 m_{\nu_\tau}}{64\pi v_1^2}. \tag{18}$$

For $m_{\nu_\tau} = 3$ MeV, the ν_τ lifetime is then about 1.3×10^4 seconds, which is within the required range for a successful explanation of the COBE data in the case of 100% cold dark matter.[11] This is a remarkable correlation between the constraint of cosmology and that of solar and atmospheric neutrino data.

The singlet neutrino ν_S is not inert, but because of the discrete Z_3 symmetry, its only interaction at tree level with the other leptons is given by $f_{\tau S} \overline{\tau}_R \nu_S \eta_0^- + H.c.$ Hence its effect on all known leptonic processes is easily shown to be negligible for $f_{\tau S} = 0.03$ and $m_\eta = 1$ TeV. It decouples from other light particles in the early Universe when the τ does. Hence its contribution to the energy density at the time of nucleosynthesis is also negligible. Since m_{12} is of order $10^{-10}\mathrm{eV}^2$, the oscillation time between ν_e and ν_S is about 10^2 seconds. This is long enough also for ν_S not to be a factor in nucleosynthesis. In fact, the contributing light degrees of freedom in this model, not counting the photon, consists of only ν_e, ν_μ, and the Majoron. Hence the effective number of neutrinos N_ν is only 2.6, below the standard upper bound of 3.3[27] or the more recently proposed 3.04[28].

Since η_2 couples to the leptons via the interactions $(\nu_e \tau_L - e_L \nu_\tau)\eta_2^+$ and $(\nu_\mu \tau_L - \mu_L \nu_\tau)\eta_2^+$, there are additional contributions to leptonic processes. For example, $\mu \to e \overline{\nu}_e \nu_\mu$ decay is accompanied by $\mu \to e \overline{\nu}_\tau \nu_\tau$ but the latter is only of order $10^{-6} G_F$ in strength. Similarly, $\mu \to e\gamma$ and $\nu_\tau \to \overline{\nu}_e \gamma + \overline{\nu}_\mu \gamma$ have branching fractions of order 10^{-14}, and $\nu_\tau \to e^- e^+ \overline{\nu}_e$ is even more negligible. Hence the standard low-energy weak-interaction phenomenology is not affected. A second comment involves CP nonconservation. In the above, since only one N_R is assumed, the ν_τ Yukawa coupling to ϕ_1^0 can be chosen real. Nevertheless, CP nonconserving couplings do exist in the Higgs sector which may or may not be sufficient for electroweak baryogenesis. If not, an easy remedy is to add one more N_R, then a CP nonconserving phase will show up explicitly in the ν_τ Yukawa coupling.

In conclusion, it has been shown in this paper that a simple ansatz for the neutrino mass

matrix, *i.e.* $\mathcal{M}_\nu$ of Eq. (1), works very well as an explanation of the present observed solar and atmospheric neutrino deficits. It is also naturally realized by a radiative mechanism based on the spontaneous breaking of lepton number. This has the advantage of incorporating electroweak baryogenesis and allowing the missing mass of the Universe to be all cold dark matter. The key is for ν_τ to be a few MeV in mass and to decay late enough to delay the ultimate time of matter-radiation equality in the early Universe. This has been accomplished in a previously proposed doublet Majoron model,[10, 11] which is now extended to include a singlet neutrino ν_{SL} with $L = 1$ and three charged scalar singlets together with a softly broken discrete Z_3 symmetry, resulting in an effective $\mathcal{M}_\nu$ exactly of the right form. Because of the necessity of maximum mixing, only the vacuum oscillation solution of the solar neutrino deficit is applicable in this scenario. However, the numbers turn out to be just right for the ν_τ lifetime. Specifically, $m \simeq 0.1$ eV from the atmospheric data, $mm'^2/m_{\nu_\tau} \simeq 10^{-10}\mathrm{eV}^2$ from the solar data, and $m_{\nu_\tau} \sim$ few MeV, $m'/m_{\nu_\tau} \sim 10^{-8}$ from cosmology.

Note Added. If there are no neutrinos beyond ν_e, ν_μ, and ν_τ, it is still possible to obtain $\mathcal{M}_\nu$ of Eq. (1) radiatively. Since a Majoron is not required, lepton number will now be assumed to be broken by explicit soft terms. In particular, the cubic term $\eta_2^- \phi_0^+ \phi_1^0$ is allowed. Hence the $\nu_e - \nu_\tau$ and $\nu_\mu - \nu_\tau$ entries are radiatively generated in one loop, but the other entries remain zero. Now let there be a doubly charged singlet scalar σ^{--} with lepton number $L = 2$ and which transforms as ω under Z_3, then the interaction $\sigma^{++}\tau_R\tau_R$ is allowed (but not with τ_R replaced by e_R or μ_R). Let there also be the cubic term $\sigma^{++}\eta_2^-\eta_2^-$ which breaks both L and Z_3, then these other entries also become nonzero in two loops.[17] Hence the desired form of the 3×3 $\mathcal{M}_\nu$ is again realized radiatively.

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FIGURE CAPTION

Fig. 1. One-loop radiative $\nu_e - \nu_s$ mass due to the spontaneous breaking of lepton number.